

OpenMath: representing mathematical meaning

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Representing the meaning of mathematics

- ▶ In a way that's "sufficiently natural"
- ▶ Suitable for automatic reasoning
- ▶ Allows for different levels of tools

Three possible routes

It's a solved problem

Use Zermelo–Fraenkel (or whatever your favourite formalism is)

- ▶ Hardly natural — a function is a set of ordered pairs, and (a, b) is $\{a, \{a, b\}\}$
- ▶ Hardly suitable for reasoning, \sin is an uncountable set
- ▶ Not relevant to the tools *most* mathematicians use.

“We know what we mean”

Largely (Content) MathML 1 and 2.

Whenever anyone says “you know what I mean”, you can be pretty sure that *he* does not know what he means, for if he did, he would tell you. — H. Davenport (1907–1969)

But do we?

- ▶ Is D_6 a group on six points or a group with six elements?
- ▶ See Conway *et al.* *LMS J. Computation and Math.* **1** (1998) pp. 1–8.
- ▶ “Well known sin” pretty much works, **but**
- ▶ “Well known arccot” does not:

$$\operatorname{arccot}(-1) = \begin{cases} 3\pi/4 & \text{Abramowitz \& Stegun 1st printing} \\ -\pi/4 & \text{Abramowitz \& Stegun 10th printing} \end{cases}$$

- ▶ Is arcsin single-valued and Arcsin multivalued?
- ▶ or the other way round (as in France)?

The OpenMath route

Define (fully or partially) as far as possible.

- ▶ plus is not defined at all
- ▶ arccot is fully defined in terms of log etc.
- ▶ Some symbols have mixed definitions, e.g. `oriented_interval` (needed to make sense of $\int_a^b f = \int_{(a,b)} f$).

Definitions live in OpenMath Content Dictionaries, *not* in the Standard itself, so OpenMath is a growing definition.